Composite Sandwich Structure Design Requirements

Composite Engineer's Viewpoint
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Panel Bending Behaviour

Sandwich panel bending behaviour is somewhat difficult based on the mathematical complexities of the analysis. Simplified solutions to the structural performance equations of motion have been derived by a number of authors and an application of that analysis is summarised here.

The deformation of a rectangular sandwich panel, Figure 1, where the core is at least four times the skin thickness, is based on specially orthotropic composite facings with the panel simply supported on all four edges under a uniformly distributed pressure load (p_o). A specially orthotropic laminate possesses mid-plane through-the-thickness symmetry has zero, or near-zero, values for the twist-coupling flexural stiffness coefficients (D_{16}, D_{61}, D_{26}, D_{62}). The deformation, based on the double Fourier sinewave series, at any point in the panel, is given by:

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi x}{b}\right)$$

$$A_{mn} = B_{mn}\left[D_{11}\left(\frac{m\pi}{a}\right)^4 + 2(D_{12}+2D_{66})\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + D_{22}\left(\frac{n\pi}{b}\right)^4\right]^{-1}$$

$$B_{mn} = \frac{4p_o}{\pi^2 mn}\left[1-(-1)^m\right]\left[1-(-1)^n\right]$$

Where:
m and n are series integer values 1, 2, 3, 4 ….
a and b are the panel length and width dimensions, respectively
D_{ij} are the 9 components of the bending stiffness matrix (i and j = 1, 2, 6)

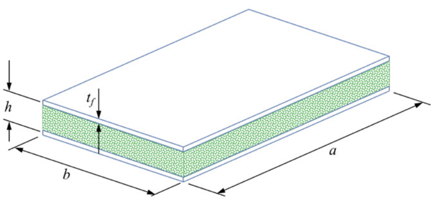

Figure 1: Sandwich Panel Geometry

The maximum deformation of the panel is at the panel mid-point, $x = a/2$ and $y = b/2$. From the double Fourier series expression of the deformation expression $w(x, y)$, even number values of m and n are zero. Typically the first four series values represent no less than 95% of the maximum deformation and for this discussion this will be considered as the limits of the summation. Thus the deformation parameters B_{mn} are:

$$B_{11} = \frac{16p_o}{\pi^2} \qquad B_{31} = B_{13} = \frac{16p_o}{3\pi^2} \qquad B_{33} = \frac{16p_o}{9\pi^2}$$

The maximum deformation at the panel centre is thus:

$$w\left(a/2, b/2\right) \approx A_{11} + A_{31} + A_{13} + A_{33}$$

Where:

$$A_{11} = \frac{16a^4 p_o}{\pi^6}\left[D_{11} + 2(D_{12}+2D_{66})R^2 + D_{22}R^4\right]^{-1}$$

$$A_{31} = \frac{16a^4 p_o}{3\pi^6}\left[81D_{11} + 18(D_{12}+2D_{66})R^2 + D_{22}R^4\right]^{-1}$$

$$A_{13} = \frac{16a^4 p_o}{3\pi^6}\left[D_{11} + 18(D_{12}+2D_{66})R^2 + 81D_{22}R^4\right]^{-1}$$

$$A_{33} = \frac{16a^4 p_o}{9\pi^6}\left[81D_{11} + 162(D_{12}+2D_{66})R^2 + 81D_{22}R^4\right]^{-1}$$

$R=a/b$ or the panel aspect ratio.

In a previous article we saw that the relationship between the bending stiffness of sandwich panels and the axial stiffness was related by $h^2/4$. With a simple parameterisation of the deflection expression we can observe the central deformation in normalised form and in terms of the skin engineering stiffness properties and panel geometry. This is expressed as:

$$w_{max}\left[\frac{9\pi^6 h^2 t_f E_{f_2}}{32a^4(1-\upsilon_{21}\upsilon_{12})p_o}\right] = \left\{\begin{array}{l} 9\left[\left(E_{f_1}/E_{f_2}\right) + 2\left(\mu_{21}\left(E_{f_1}/E_{f_2}\right) + \left(G_{f_{12}}/E_{f_2}\right)\right)R^2 + R^4\right]^{-1} \\ -3\left[81\left(E_{f_1}/E_{f_2}\right) + 18\left(\mu_{21}\left(E_{f_1}/E_{f_2}\right) + \left(G_{f_{12}}/E_{f_2}\right)\right)R^2 + E_{f_2}R^4\right]^{-1} \\ -3\left[\left(E_{f_1}/E_{f_2}\right) + 18\left(\mu_{21}\left(E_{f_1}/E_{f_2}\right) + \left(G_{f_{12}}/E_{f_2}\right)\right)R^2 + 81E_{f_2}R^4\right]^{-1} \\ +\left[81\left(E_{f_1}/E_{f_2}\right) + 162\left(\mu_{21}\left(E_{f_1}/E_{f_2}\right) + \left(G_{f_{12}}/E_{f_2}\right)\right)R^2 + 81E_{f_2}R^4\right]^{-1} \end{array}\right\}$$

Whilst geometry (aspect ratio), boundary conditions and surface load all play an important role in determining the panel bending performance, the composite skinned sandwich structure flexural stiffness [Dij] has a role to play in structural performance, as seen in Figure 2.

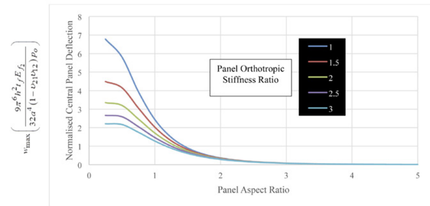

Figure 2: Central Bending Deformation of a Composite Skinned Sandwich Panel

This normalised plot (Figure 2) clearly shows that as the aspect ratio of the sandwich panel increases the influence of the composite skin laminate configuration becomes insignificant and, at aspect ratios of two and greater the panel will perform like a sandwich structure with isotropic skins. Also note that core shear deformation is neglected if the panel planar dimensions are greater that 50 times the core thickness. Other boundary conditions (fixed or clamped) will decrease the panel bending deformation performance.

In the next article we will discuss sandwich panel vibration behaviour with a focus on natural frequency determination and the effects of damping afforded by composite materials.

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